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Localization Effects and Dispersion of the Dielectric Response Function in One-Dimensional Metals

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We report the results of calculations and analysis concerning the dependence of the dielectric response function $\varepsilon(\omega, q)$ on frequency ω and wave number q in one-dimensional conductors. The localization of electron states leads to unusually complicated dependences of $\varepsilon(\omega, q)$ on ω and q in low-frequency and long-wavelength regions, while the spatial and time dispersions become closely interwoven with each other. The effect of geometric resonance is discussed. It appears as quasiharmonical oscillations of complex susceptibility as function of ω and q, owing to the hopping nature of electron conductivity in a nonuniform electromagnetic field.

KEY WORDS: One-dimensional disordered systems; one-dimensional conductor; the localization of electron states; dielectric response function; time and space dispersion.

1. INTRODUCTION

The basic feature of a perfect crystal is that the atoms regularly disposed at lattice sites do not scatter electrons in definite energy regions, which is due to the quantum nature of electronic motion. In such allowed energy bands, the electronic wave functions, though modified with respect to the plane waves, yet extend over the whole of a crystal. In other words, the probability of finding the quasiparticle in any volume element of a perfect crystal is a nonzero constant. The fact that random lattice irregularities in real crystals give rise to the scattering of quasiparticles had been realized a long time ago. I. Lifshitz was the first to point out the possibility and to give conditions for appearance of a new kind of quasiparticle states which are localized near the

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breakings of translational symmetry in a crystal. He developed the mathematical approach for description of such localized states known as "the regular disturbance method."[1] This method proved to be so effective as to enable even studying surface vibrations, while taking the free boundary of the crystal as the violation of its spatial homogeneity.[2] These papers have laid the foundations of the wide region in solid state physics that is investigations of disordered systems. The results obtained in this field by I. Lifshitz and his co-workers and students have recently been summarized in a monograph[3] along with the review of the current state of the disordered system physics.

The existence of a macroscopic number of localized states is the most distinguishing feature of disordered systems. For electrons in metals this phenomenon has been revealed by Anderson.[4] The localization affects most pronouncedly the properties of one-dimensional metals where all the electronic states become localized in arbitrarily small random potential.[5] This is due to anomalously large fluctuations of phases in the wave functions of electrons moving in one dimension. A consistent approach incorporating this physical idea in adequate mathematical form was represented in a book[3] by I. Lifshitz *et al.*

The transformation of propagating waves to localized wave functions of electrons causes, in the first place, the changes in kinetic characteristics. One of the most important of them is the complex dielectric response function

$$\varepsilon(\omega, q) = \varepsilon'(\omega, q) + i\varepsilon''(\omega, q) \tag{1.1}$$

which is connected with the polarizability $\alpha(\omega, q)$ and dissipative conductivity $\sigma(\omega, q)$ by the following relationships:

$$\varepsilon'(\omega, q) = 1 + 4\pi\alpha(\omega, q), \qquad \varepsilon''(\omega, q) = \frac{4\pi}{\omega}\sigma(\omega, q) \tag{1.2}$$

Here ω and q are, respectively, the frequency and wave number of an external electromagnetic field.

The localization effects influence essentially the time and spatial dispersions of the response function. Among all the variety of problems arising thereby, only the ω dependence of $\varepsilon(\omega)$ at $q = 0$, that is in a uniform alternating field, has been so far investigated in detail. In this case, the low-frequency conductivity $\sigma(\omega)$ was qualitatively analyzed by Mott.[6] He pointed out that $\sigma(\omega) \sim \omega^2 \ln^2 \omega$ with $\omega \to 0$. This kind of low-frequency behavior of $\sigma(\omega)$ has been confirmed by Berezinsky[7] within the frame of rigorous approach based on the direct summation of perturbation theory expansions. Using Berezinsky's equations, in Ref. 8 the numerical calculations of $\varepsilon'(\omega)$ and $\sigma(\omega)$ have been performed with arbitrary values

of ω. The maximum has been found which $\sigma(\omega)$ has at $\omega\tau \simeq 0.62$, τ being the mean free time of electron with respect to the backscattering. Near this frequency $\varepsilon'(\omega)$ changes its sign. The temperature dependence of the static conductivity $\sigma(0)$ caused by scattering on phonons has been discussed by many authors. All the results obtained are reviewed by Gogolin.[9]

It should be stressed, however, that not the time dispersion of the response function, but the spatial one turns out to be most susceptible to the structure and geometrical characteristics of localized wave functions. In principle, Berezinsky's equations allow one to find the dependence of $\varepsilon(\omega, q)$ on q. Nonetheless, the spatial dispersion of $\varepsilon(\omega, q)$ was not studied up until recently because considerable mathematical difficulties arise.

In this paper we will examine the dispersion of the dielectric response function $\varepsilon(\omega, q)$ throughout the whole range of ω's and q's, at zero temperature, using Berezinsky's equations. In the next section the basic relationships are adduced which make up a starting point for calculating the polarizability and conductivity in the low-frequency limit; also the effect of fluctuations of the relaxation time in the one-dimensional system is discussed which has been predicted by one of the authors.[10] In the third section we analyze the effect of geometrical resonance in dissipative conductivity,[11] which is due to the hopping nature of the conductivity with a fixed length of jumps in a nonuniform field of external wave. In the final section the high-frequency dielectric susceptibility is investigated on the basis of the general formula for $\varepsilon(\omega, q)$, being valid at any frequencies and wave numbers.

2. LOW–FREQUENCY DIELECTRIC SUSCEPTIBILITY

The dielectric response function in a one-dimensional with allowance for the time and spatial dispersions can be calculated, according to Ref. 7, with the formula

$$\varepsilon(\omega, q) = 1 + (\omega_p \tau)^2 \int_0^\infty dt\, e^{-xt}[y(t, q) + y(t, -q)] \qquad x = -2i\omega\tau \tag{2.1}$$

Here ω_p is Lengmuir's frequency of electrons, τ is the mean free time of electron with respect to the backscattering. The function $y(t, q)$ is the solution of the equation

$$-\frac{d}{dt}\left[t(1+t)\frac{dy}{dt}\right] + xt\frac{d}{dt}[(1+t)y] - i(\omega - qv)\tau y = -xe^{x(1+t)}\,\mathrm{Ei}[-x(1+t)] \tag{2.2}$$

considered on the semiaxis $0 \leqslant t < \infty$ along with the boundary conditions which demand $y(t)$ to be finite at $t = 0$ and vanishing with $t \to +\infty$. Ei(z) is

the integral exponential function, v is the Fermi velocity. Equations (2.1) and (2.2) enable us to find the dispersion of $\varepsilon(\omega, q)$ at arbitrary ω and q. But so far Eq. (2.2) has been solved explicitly only in the limiting cases of low or high frequencies. Here we will consider the dispersion of $\varepsilon(\omega, q)$ in the low-frequency region, $\omega\tau \ll 1$.

Because of the localization, a one-dimensional system of electrons behaves like an insulator where the real part of $\varepsilon(\omega, q)$ is large compared with its imaginary part. Therefore, let us calculate first $\varepsilon'(\omega, q)$, which is determined by the values of $y(t)$ with large arguments, $t \sim 1/x$. Introduce a new variable $\xi = x(1+t)$ and rewrite Eq. (2.2) in terms of ξ:

$$-\frac{d}{d\xi}\left(\xi^2 \frac{dy}{d\xi}\right) + \xi \frac{d}{d\xi}(\xi y) + x \frac{d}{d\xi}\left[\xi\left(\frac{dy}{d\xi} - y\right)\right] - i(\omega - qv)\tau y = xe^{\xi} \operatorname{Ei}(-\xi) \tag{2.3}$$

In the main approximation we put $x = 0$ in the left-hand side of Eq. (2.3) and find the exact solution $y_0(\xi)$ of the remaining equation:

$$y_0(\xi) = \frac{x}{2}\left(\frac{\pi}{\xi}\right)^{1/2} e^{\xi/2} \int_0^\infty d\mu \, \frac{\mu \operatorname{sh}(\pi\mu/2)}{\operatorname{ch}^2(\pi\mu/2)} \frac{K_{i\mu/2}(\xi/2)}{\nu(\mu) + iql} \tag{2.4}$$

where $l = v\tau$ is the mean free path, $K_{i\mu}(z)$ is Macdonald's function with imaginary index, $\nu(\mu) = (1+\mu^2)/4$. Inserting Eq. (2.4) into (2.1), we get

$$\varepsilon'(\omega, q) = 1 + 2(\omega_p \tau)^2 \int_0^\infty d\mu \, w_0(\mu) \frac{\nu(\mu)}{\nu^2(\mu) + (ql)^2} \tag{2.5}$$

$$w_0(\mu) = \frac{\pi^2}{2} \frac{\mu \operatorname{sh}(\pi\mu/2)}{\operatorname{ch}^3(\pi\mu/2)} \tag{2.6}$$

The structure of Eqs. (2.4) and (2.5) points out that $\nu(\mu)$ has the sense of the dimensionless (measured in units of $1/\tau$) collision frequency of electrons scattered by impurities. The value $\nu(\mu)$ is a real and positive eigenvalue of an operator determined by the two first terms in the left-hand side of Eq. (2.3). Hence, this operator may be interpreted as a quantum-mechanical operator of the collision frequency. From this viewpoint, the integration over μ in Eq. (2.5) means averaging the energy denominators $[\nu(\mu) \pm iql]^{-1}$ over different values of the collision frequency $\nu(\mu)$, with the probability distribution function $w_0(\mu)$ for the "quantum numbers" $\mu > 0$. While positive and normalized, the function $w_0(\mu)$ really has the sense of a distribution function. It should be stressed however, that the averaging over μ is carried out at a fixed quantum state of the electron, so it in principle cannot be treated as averaging over diverse electron states.

Consequently, the result obtained, (2.5), shows that in a one-dimensional system the transport collision frequency for an electron having fixed momentum and spin does not have any definite value, but fluctuates like a dynamical variable in the quantum mechanics.

The nature of the collision frequency fluctuations can be understood by the following physical considerations. The localization of electron states in a one-dimensional system arises due to a finite probability for an electron to be scattered backwards, which is equal to $1/\tau$. In other words, an electron moves in one direction during the time τ and then is scattered in the opposite direction. For various electrons these times should be different just because it is only a *probability* of the backscattering that is given. The diversity in the scattering times is the very thing to cause the fluctuations of the relaxation frequency. These fluctuations have a phase origin, since the finite relaxation time arises as a result of the fluctuations of phases in wave functions.[12] It is just for this reason that the fluctuations of the collision frequency have a quantum nature.[10]

In Eq. (2.5), the leading term in the asymptotic expansion of $\varepsilon'(\omega, q)$ does not depend on ω, but includes the spatial dispersion effects. In the limiting cases of small or large ql we get:

$$\varepsilon'(\omega, q) = 1 + 4\zeta(3)(\omega_p\tau)^2, \qquad (ql)^2 \ll 1 \tag{2.7a}$$

$$\varepsilon'(\omega, q) = 1 + \frac{(\omega_p\tau)^2}{(ql)^2}, \qquad (ql)^2 \gg 1 \tag{2.7b}$$

where $\zeta(z)$ is the ζ-function of Riemann. The expression (2.7a) for $\varepsilon'(0, 0)$ was first obtained in Ref. 13.

In the next approximation in $\omega\tau$ there appear both a nonzero imaginary (dissipative) part $\varepsilon''(\omega, q)$ and the frequency dispersion of $\varepsilon'(\omega, q)$. In order to calculate $\varepsilon''(\omega, q)$, it is sufficient to drop the second term in the left-hand side of Eq. (2.2), since the main contribution in the integral (2.1) comes from large t's, $t \ll 1/x$. After this is done, we arrive at an equation which can be solved exactly. Denoting this solution $y_1(t)$, we find

$$y_1(t) = \frac{(\pi x)^{1/2}}{2} e^{x/2} \int_0^\infty d\mu \, \frac{\mu \operatorname{sh}(\pi\mu/2)}{\operatorname{ch}^2(\pi\mu/2)} \frac{K_{i\mu/2}(x/2)}{\nu(\mu) + iql} P_{-(1+i\mu)/2}(1 + 2t) \tag{2.8}$$

where $P_\nu(z)$ is Legendre's function. Inserting Eq. (2.8) into Eq. (2.1) and calculating integrals, we obtain at $ql \ll 1$:

$$\varepsilon''(\omega, q) = 2(\omega_p\tau)^2 \frac{(\omega\tau)}{(ql)^2} \left\{ 1 - (\omega\tau/2)^{2(ql)^2} \left[\cos\left(2ql \ln \frac{\omega\tau}{2} \right) - 2ql(\ln 4 + C - 1) \sin\left(2ql \ln \frac{\omega\tau}{2} \right) \right] \right\} \tag{2.9}$$

where $C = 0.577...$ is Euler's constant. The expression (2.9) exhibits unusually complicated dependence of $\varepsilon''(\omega, q)$ on ω and q in the region of low frequencies and long wavelengths. Here the time and spatial dispersions of $\varepsilon''(\omega, q)$ become whimsically entangled.

If $2ql\,|\ln(\omega\tau/2)| \ll 1$, Eq. (2.9) gives Berezinsky's result:

$$\varepsilon''(\omega, q) = 4(\omega_p\tau)^2(\omega\tau)\{\ln^2(2\omega\tau) + (2C - 3)\ln(2\omega\tau) + \text{const}\} \quad (2.10)$$

At $ql \gg 1$ we find for the dissipative part of $\varepsilon(\omega, q)$ the expression

$$\varepsilon''(\omega, q) = 2(\omega_p\tau)^2 \frac{\omega\tau}{(ql)^4} \quad (2.11)$$

In this region of ql, $\varepsilon''(\omega, q)$ is proportional to $1/\tau$. This means that the collisionless Landau damping is absent, so the dissipative mechanism is connected with electron-impurity collisions.

The same function $y_1(t)$ allows us to find the frequency dependence of the real part $\varepsilon'(\omega, q)$. At small ql, $ql \ll 1$, the correction for (2.7a) is found with a logarithmic accuracy:

$$\delta\varepsilon'(\omega, q) = 2\pi(\omega_p\tau)^2 \frac{\omega\tau}{ql} (\omega\tau/2)^{2(ql)^2} \sin\left(2ql \ln\frac{\omega\tau}{2}\right) \quad (2.12)$$

In particular, when ql is so small that $2ql\,|\ln(\omega\tau/2)| \ll 1$ and $\varepsilon''(\omega, q)$ is given by Eq. (2.10), the real part of $\varepsilon(\omega, q)$ is

$$\varepsilon'(\omega, q) = 1 + 4(\omega_p\tau)^2[\zeta(3) + \pi\omega\tau \ln(\omega\tau) + O(\omega\tau)] \quad (2.13)$$

The ω-dependent correction for Eq. (2.7b) at $ql \gg 1$ has a relatively small $(\omega\tau)^2$ compared to the second term in (2.7b), so it will not be written here.

3. THE EFFECT OF GEOMETRIC RESONANCE

The formulas (2.9) and (2.12) obtained in the previous section show that both imaginary and real parts of the dielectric response function oscillate as functions of frequency ω or wave number q. These oscillations are exhibited in the region $\kappa \ll 1$,

$$\kappa^2 L_\omega < 1 < \kappa L_\omega, \qquad \kappa = ql, \qquad L_\omega = 2\ln(2/\omega\tau) \quad (3.1)$$

The dissipative conductivity $\sigma(\omega, q)$ in this region, according to (2.9) and (1.2), is

$$\frac{\sigma(\omega, q)}{\sigma_0} = 4(\omega\tau)^2 \left[\frac{1}{\kappa^2} \sin^2\left(\frac{\kappa L_\omega}{2}\right) + \frac{L_\omega}{2}\right], \qquad \sigma_0 = \frac{ne^2\tau}{m} \quad (3.2)$$

The oscillation's period in κ is $2\pi/L_\omega$, the relative amplitude is of order $2/\kappa^2 L_\omega$, while the minimum value is $2(\omega\tau)^2 L_\omega$. In the range $1/L_\omega \lesssim \kappa^2 < 1$, the oscillating terms become exponentially small, so $\sigma(\omega, q) = 2(\omega\tau/\kappa)^2 \sigma_0$.

Let us dwell on the physical nature of these oscillations. Equation (3.2) reveals a characteristic electronic length $2l\,|\ln(\omega\tau/2)|$ which determines the oscillation's period in a nonuniform field of external wave. It easily seen that this length may be taken as the fixed (at given ω) length of an electron's jump between two localized states having energy levels separated from each other by a quantum ω. Indeed, in accordance with Ref. 12, the low-frequency conductivity can be estimated by the order of magnitude, following the formula

$$\frac{\sigma(\omega, q)}{\sigma_0} \simeq |D(\omega, q)|^2 \tag{3.3}$$

Here $D(\omega, q)$ is the dimensionless Fourier transform of the velocity's matrix element:

$$D(\omega, q) = \frac{1}{2i}\int_{-\infty}^{\infty} d\xi\, \psi_{\varepsilon_F+\omega}(\xi)\left(e^{i\kappa\xi}\frac{d}{d\xi} + \frac{d}{d\xi}e^{i\kappa\xi}\right)\psi_{\varepsilon_F}(\xi) \tag{3.4}$$

calculated between the wave functions of localized states having energies ε_F and $\varepsilon_F + \omega$ (ε_F is the Fermi energy):

$$\psi_{\varepsilon_F}(\xi) = \exp(-|\xi|/2), \qquad \psi_{\varepsilon_F+\omega}(\xi) = \exp(-|\xi - L_\omega|/2) \tag{3.5}$$

The maxima of these functions are set apart at a distance L_ω from each other because their energy difference is ω, $\omega \ll 1/\tau$.[12] The main contribution in the integral (3.4) at small κ gives the interval $0 < \xi < L_\omega$, so we find

$$|D(\omega, q)|^2 = e^{-L_\omega}\frac{1}{\kappa^2}\sin^2\left(\frac{\kappa L_\omega}{2}\right) = \left(\frac{\omega}{2qv}\right)^2 \sin^2\left(ql\ln\frac{\omega\tau}{2}\right) \tag{3.6}$$

This result coincides with the first two terms in Eq. (2.9) to an accuracy of a numerical factor.

Hence, the quasiharmonical oscillations in both the conductivity $\sigma(\omega, q)$ and the susceptibility $\varepsilon'(\omega, q)$ are the effect of the geometric resonance's type, predicted in Ref. 11. Much like the geometric resonance in the sound attenuation in a magnetic field in metals,[14] the oscillations found result from the nonmonotonic dependence of the overlap integral on the phase increment which arises when an electron jumps over the distance $2l\,|\ln(\omega\tau/2)|$. Note that the correct result (3.6) can be obtained only provided the wave function of a localized state has a kink near its maximum. Thus

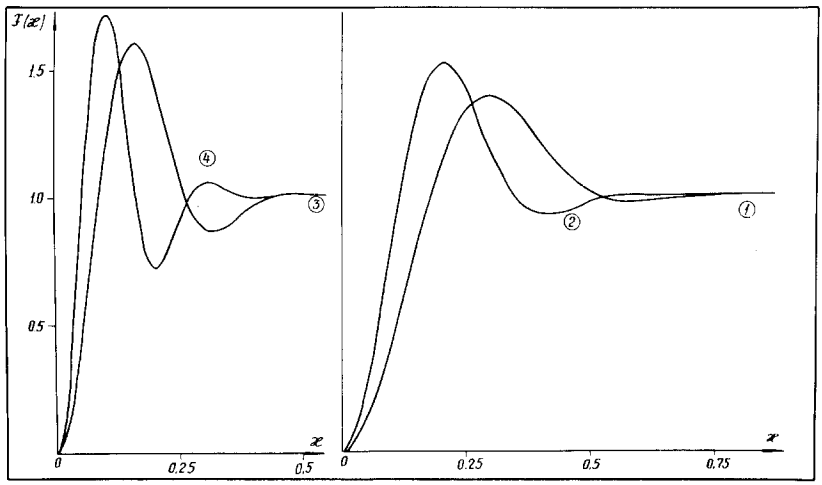

Fig. 1. The dependence of $\mathcal{F}(\kappa)$ on wave number q with various $\omega\tau$: 1, 10^{-2}; 2, 10^{-3}; 3, 10^{-4}; 4, 10^{-6}.

one should think that the wave function (3.5) giving the geometric oscillations (2.9). reproduces correctly the structure of a localized electron state.

The calculated dependence of the function $\mathcal{F}(\kappa)$ given by the expression in brackets of Eq. (2.9) is plotted in Fig. 1 at various $\omega\tau$. The first oscillation is prominently visible, while the second one appears with sufficiently small $\omega\tau$. The further periods cannot be seen, for they are located at ql such that the amplitude of the oscillating term becomes negligibly small.

4. HIGH-FREQUENCY DISPERSION OF THE SUSCEPTIBILITY

While analyzing representations for $\varepsilon(\omega, q)$ in various limits, we arrive at a conclusion that the expression for the dielectric response function in a one-dimensional conductor should have at any ω and q the form

$$\varepsilon(\omega, q) = 1 + (\omega_p\tau)^2 \sum_{\pm} \frac{1}{2} \int_0^\infty d\mu\, w_\infty(\mu) \frac{iD_\mu(x)/\omega\tau}{\nu(\mu) - i(\omega \pm qv)\tau} \tag{4.1}$$

where $w_\infty(\mu)$ denotes the function

$$w_\infty(\mu) = \frac{\pi}{2} \frac{\mu\, \mathrm{sh}(\pi\mu/2)}{\mathrm{ch}^2(\pi\mu/2)} \tag{4.2}$$

The summation in (4.1) is over the electronic states on the Fermi surface of one-dimensional conductor $\pm p_0$ with the momentum distribution function, which is equal to 1/2.

The function $D_\mu(x)$ describes the influence of localization of electron states on the complex conduction. We call $D_\mu(x)$ the "delocalization factor." Taking advantage of the results in Sec. 2, we can find the low-frequency behavior of $D_\mu(x)$, $|x| \ll 1$:

$$D_\mu(x) = \frac{\pi x}{\operatorname{ch}\left(\frac{\pi\mu}{2}\right)} - \frac{\pi x}{4\operatorname{sh}^2\left(\frac{\pi\mu}{2}\right)} \times \left[\Gamma^{-2}\left(1+\frac{i\mu}{2}\right)\left(\frac{x}{4}\right)^{i\mu} + \Gamma^{-2}\left(1-\frac{i\mu}{2}\right)\left(\frac{x}{4}\right)^{-i\mu}\right] + O(x^2) \quad (4.3)$$

The vanishing of the delocalization factor with $x \to 0$ gives evidence of the known fact that in this limit the localization destructively affects the conductivity.[3]

In the high-frequency limit ($|x| \to \infty$), $D_\mu(x)$ goes to unity, which means switching off the localization effect in conductivity:

$$D_\mu(x) = 1 - \frac{2\nu(\mu)}{x} + O\left(\frac{1}{x^2}\right) \quad (4.4)$$

Thus, the formula (4.1) shows that the susceptibility $\varepsilon(\omega, q)$ of the one-dimensional conductor is determined by the combined influence of the two different effects, which are the localization of electron states, and the fluctuations of the collision frequency of electrons. The coupling of these two factors is manifested in that the function $D_\mu(x)$ depends both on the frequency (x) and on μ.

In the high-frequency limit, $\omega\tau \gg 1$, from Eq. (4.1) and (4.4) we get

$$\varepsilon(\omega, q) = 1 + (\omega_p\tau)^2 \int_0^\infty d\mu \frac{w_\infty(\mu)}{[\nu(\mu) - i\omega\tau]^2 + (ql)^2}\left[1 + O\left(\frac{1}{\omega^2\tau^2}\right)\right] \quad (4.5)$$

Integration over μ retains even in this case, as before, the sense of averaging over the fluctuations of dimensionless collision frequency with the probability distribution function $w_\infty(\mu)$, written in Eq. (4.2). The expression for $w_\infty(\mu)$ differs only slightly from the function $w_0(\mu)$ describing the collision frequency distribution in the low-frequency limit.

The function (4.2) has the following properties:

$$\int_0^\infty d\mu\, w_\infty(\mu) = 1, \qquad \int_0^\infty d\mu\, w_\infty(\mu)\,\nu(\mu) = 1 \quad (4.6)$$

Equation (4.5) shows that in the main approximation in $\omega\tau$, when one can drop $\nu(\mu)$ in the denominator, the real part of $\varepsilon(\omega, q)$ is given by Drude's formula:

$$\varepsilon'(\omega, q) = 1 + \frac{\omega_p^2}{(qv)^2 - \omega^2} \tag{4.7}$$

This expression fails in the resonance region, where

$$|\omega - qv| \lesssim \frac{1}{\tau} \tag{4.8}$$

and one should retain the relatively small quantity $\nu(\mu)$ in the denominator of integrand (4.5). Hence, near this resonance the collision frequency's fluctuations can become revealed.

As for $\operatorname{Im} \varepsilon(\omega, q)$, Drude's formula gives an incorrect (half as much) numerical factor in the region of weak spatial dispersion, $ql \ll 1$, and the wrong dependence on ω and q if $qv \gg \omega$.

Outside the resonant region (4.8), Eq. (4.5) leads to the expression

$$\varepsilon''(\omega, q) = \frac{1}{\tau} \frac{2\omega_p^2 \omega}{(q^2 v^2 - \omega^2)^2} \tag{4.9}$$

It is seen that at $qv \gg \omega$ Eq. (4.9) is transformed to Eq. (2.11). This means that the condition for the low-frequency asymptotics (2.7b) and (2.11) to hold at large ql is $(ql)^2 \gg 1 + (\omega\tau)^2$, whatever $\omega\tau$ may be.

It should be emphasized that the expression for $\varepsilon''(\omega, q)$ including spatial dispersion effects cannot be obtained using conventional kinetic equation for electrons in one-dimensional metals. The results for $\varepsilon''(\omega, q)$ can be qualitatively reproduced in the high-frequency region provided the localized electrons are considered as charged oscillators having characteristic natural frequency of the order $1/\tau$.

The formulas (4.7) and (4.9) enable us to find the spectrum and damping of longitudinal plasma waves in one-dimensional metals.

When the wave vector $\mathbf{q}$ makes an arbitrary angle α with conducting chaines, the expressions (4.7) and (4.9) still hold provided one puts $\omega_p \to \omega_p \cos\alpha$, $qv \to qv \cos\alpha$. So the frequency of Lengmuir's plasmon proves to be

$$\omega(\mathbf{q}) = \omega_p |\cos\alpha| \left(1 + \frac{q^2 v^2}{2\omega_p^2}\right) - \frac{i}{\tau} \tag{4.10}$$

It is weakly damped if $|\cos\alpha| \gg 1/\omega_p \tau$.

Let us draw attention to the specific feature of one-dimensional conductors, Landau's damping being absent in them. This is due to the fact

that the electrons on the Fermi surface are in essence two monochromatic beams with equal speeds but opposite velocities $\pm v$ [see (4.7) and (4.9)]. As a result, the averaging over electron states on the Fermi surface does not include integration over the directions of the vector **v**. As a consequence, the delta functions, $\delta(\omega \pm qv)$, describing Landau's damping do not appear. Nonetheless, spatial dispersion effects in the complex conduction of one-dimensional conductors turn out to be much more complicated than one would expect if starting from intuitive analogy with three-dimensional metals.

REFERENCES

1. I. M. Lifshitz, *Zh. Eksp. Teor. Fiz.* **17**:1017, 1076 (1947).
2. I. M. Lifshitz and L. N. Rosenzweig, *Zh. Eksp. Teor. Fiz.* **18**:1012 (1948).
3. I. M. Lifshitz, S. A. Gredescul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* (Nauka, Moscow, 1982).
4. P. W. Anderson, *Phys. Rev.* **109**:1498 (1958).
5. N. F. Mott and W. D. Twose, *Adv. Phys.* **10**:107 (1961).
6. N. F. Mott, *Adv. Phys.* **16**:49 (1967).
7. V. L. Berezinsky, *Sov. Phys. JETP* **38**:620 (1974).
8. A. A. Gogolin and V. I. Melnikov, *Phys. Stat. Sol. (b)* **88**:377 (1978).
9. A. A. Gogolin, *Phys. Rep.* **86**:2 (1982).
10. L. V. Chebotarev, *Solid State Commun.* **47**:549 (1983).
11. É. A. Kaner and L. V. Chebotarev, *Pis'ma Zh. Eksp. Teor. Fiz.* **38**:253 (1983).
12. N. F. Mott and E. Davis, *Electronic Processes in Non-Crystalline Materials* (Clarendon Press, Oxford, 1973).
13. A. A. Gogolin, V. I. Melnikov, and E. I. Rashba, *Sov. Phys. JETP* **42**:168 (1976).
14. A. B. Pippard, *Phil. Mag.* **2**:1147 (1957).